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BRIEF COMMUNICATION

FLUID VELOCITY VARIATION IN FILTER CAKES

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In the derivation of the parabolic filtrate discharge equation for the confined axial growth of a filter cake under a constant applied pressure, Ruth (1935) assumed that the superficial fluid velocity throughout the filter cake was constant at any instant. Tiller & Cooper (1960) were the first who recognized the variation of superficial fluid velocity inside the compressible filter cakes and showed that the outlet superficial fluid velocity, q_0, is always greater than the inlet superficial fluid velocity, q_i. Unfortunately, in the derivation the compaction effect was neglected and it took four years† of heuristic manipulations to obtain the correct expression (Tiller & Shirato 1964).

The derivation mentioned above is restricted to one-dimensional filtration and no attempt has been made to extend this derivation to other filter cake geometries. In fact, filter cakes of various geometrical shapes have received much less attention than the traditional cylindrically confined axial filtration although there has been some notable work done in this area by Brenner (1961), Leonard & Brenner (1965), Shirato & Kobayashi (1969), and Yoshioka, Ueda & Miyoshi (1972). The purpose of this note is to derive a general material balance which focuses on the difference between outlet and inlet cake fluid velocities and which applies to filter cakes of any geometry.

MASS INVENTORY FOR COMPRESSIBLE CAKES OF ARBITRARY GEOMETRY

For compressible filter cakes, the equation of continuity is

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot \mathbf{q} = 0 \qquad [1]$$

where ϵ is the local porosity and $\mathbf{q}$ is the superficial fluid velocity. Integration of [1] over the time-dependent filter cake volume, $V(t)$, gives

$$\int_{V(t)} \left(\frac{\partial \epsilon}{\partial t} + \nabla \cdot \mathbf{q} \right) \mathrm{d}V = 0 \qquad [2]$$

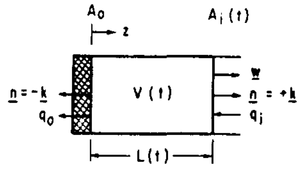
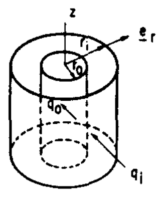
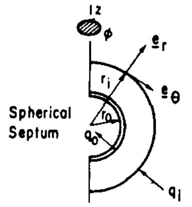
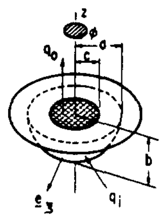
The Reynolds transport theorem and the divergence theorem permit [2] to be written as

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{V(t)} \epsilon \, \mathrm{d}V - \int_{A_i(t)} \epsilon (\mathbf{w} \cdot \mathbf{n}) \, \mathrm{d}A + \int_{A_i(t)} \mathbf{q} \cdot \mathbf{n} \, \mathrm{d}A + \int_{A_0} \mathbf{q} \cdot \mathbf{n} \, \mathrm{d}A = 0 \qquad [3]$$

where $\mathbf{w}$ is the velocity of the moving inlet cake surface $A_i(t)$ and $\mathbf{n}$ is the outwardly directed unit

†The time from which the first incorrect expression appeared to the time it was finally corrected.

Table 1. Effect of filter cake geometry on the macroscopic mass inventory equation

Filter Cake Geometry	Macroscopic Mass Inventory Equation
	$q_0 - q_i = (\epsilon_i - \epsilon^*)\frac{dL}{dt} - L\frac{d\epsilon^*}{dt}$
	$r_0 q_0 - r_i q_i = (\epsilon_i - \epsilon^*)\frac{dr_i}{dt} - \frac{1}{2}(r_i^2 - r_0^2)\frac{d\epsilon^*}{dt}$
	$r_0^2 q_0 - r_i^2 q_i = (\epsilon_i - \epsilon^*)\frac{dr_i}{dt} - \frac{1}{3}(r_i^3 - r_0^3)\frac{d\epsilon^*}{dt}$
	$q_0 - q_i\left[(1+\xi_i^2) + \xi_i^2\sqrt{1+\xi_i^2}\ \ln\left(\frac{1+\sqrt{1+\xi_i^2}}{\xi_i}\right)\right] = \frac{2c}{3}(\epsilon_i - \epsilon^*)(1+3\xi_i^2)\frac{d\xi_i}{dt} - \frac{2c}{3}(\xi_i + \xi_i^3)\frac{d\epsilon^*}{dt}$

normal vector to the surface. A_0 is the fixed area of the septum at the outlet side of the filter cake.

If the average porosity, ϵ^*, defined by

$$\epsilon^* = \frac{\int_{V(t)} \epsilon \, dV}{\int_{V(t)} dV} \tag{4}$$

and the obvious relation

$$\frac{dV}{dt} = \int_{A_i(t)} \mathbf{w} \cdot \mathbf{n} \, dA, \tag{5}$$

which can be obtained directly from the Reynolds transport theorem for an integrand of unity, are used in [3], the result is

$$\int_{A_i(t)} \mathbf{q} \cdot \mathbf{n} \, dA + \int_{A_0} \mathbf{q} \cdot \mathbf{n} \, dA = (\epsilon_i - \epsilon^*)\frac{dV}{dt} - V\frac{d\epsilon^*}{dt} \tag{6}$$

The only restriction on [6] is that the surface porosity, ϵ_i, is constant on $A_i(t)$. The two terms on the left represent the difference between the inlet and outlet flow rates, while the first term on the right hand side is proportional to the change in the cake volume and the second term on the right hand side is a compaction effect caused by changes in the average porosity.

Table 2. Effect of filter cake geometry on the functionality of porosity and velocity

	CYLINDRICAL (Radial)	SPHERICAL	OBLATE SPHEROIDAL
Transformation	$x=r\cos\theta$ $y=r\sin\theta$ $z=z$	$x=r\sin\theta\cos\phi$ $y=r\sin\theta\sin\phi$ $z=r\cos\theta$	$x=c[(1+\xi^2)(1-\eta^2)]^{1/2}\cos\phi$ $y=c[(1+\xi^2)(1-\eta^2)]^{1/2}\sin\phi$ $z=c\,\xi\eta$
Metric Coefficients	$h_r=1$ $h_\theta=1$ $h_z=1$	$h_r=1$ $h_\theta=r$ $h_\phi=r\sin\theta$	$h_\xi=c(\xi^2+\eta^2)^{1/2}/(1+\xi^2)^{1/2}$ $h_\eta=c(\xi^2+\eta^2)^{1/2}/(1-\eta^2)^{1/2}$ $h_\phi=c[(1+\xi^2)(1-\eta^2)]^{1/2}$
Cake Volume	$V=\pi h(r_i^2-r_o^2)$	$V=\frac{2}{3}\pi(r_i^3-r_o^3)$	$V=\frac{2}{3}\pi c^3(\xi_i+\xi_i^3)$
Velocity Functionality	$q_r=q_r(r,t)$	$q_r=q_r(r,t)$	$\underline{q}=q_\xi(\xi,t)\,\underline{e}_\xi(\xi,\eta)$
Porosity Functionality	$\varepsilon=\varepsilon(r,t)$	$\varepsilon=\varepsilon(r,t)$	$\varepsilon=\varepsilon(\xi,t)$

The mass balance [6] is applied to four different geometries to obtain expressions for the difference between outlet and inlet flow rates and the results are shown in table 1. In table 2, the transformations, metric coefficients, h, and general functional dependence of the superficial fluid velocity and porosity are given.

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